

CAN WE DETERMINE THE NUCLEAR EQUATION OF STATE FROM HEAVY ION COLLISIONS?

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We discuss the problems involved in extracting the nuclear equation-of-state from heavy-ion collisions. We demonstrate that the equation of state becomes effectively softer in non-equilibrium and this effect is observable in terms of collective flow effects. Thus, non-equilibrium effects must be included in transport descriptions on the level of the effective mean fields. A comparison with transverse momentum, rapidity, and centrality selected flow data show the reliability and limitations of the underlying interaction which was derived from microscopic Dirac-Brueckner (DB) results.

Heavy-ion collisions open the possibility to explore the nuclear equation-of-state (EOS) far away from saturation. In particular, the collective flow is connected to the dynamics during the high density phase of such reactions and thus yields information on the nuclear EOS ¹. The different components of collective flow, in particular, the in-plane and out-of-plane flow ² has recently attracted great interest ^{2,3,4}, since also the energy, centrality and transverse momentum dependence has been measured.

From the theoretical point of view, however, the determination of the EOS, which is an equilibrium concept, is not straightforward in heavy ion collisions, which are highly non-equilibrium processes. An appropriate starting point are the Kadanoff-Baym equations ⁵ for the non-equilibrium real-time Green functions. To arrive at transport descriptions several approximations are usually introduced: the semi-classical approximation neglecting non-locality and memory effects, the quasi-particle approximation neglecting the finite width of the spectral functions, and the T-Matrix approximation, specifying the self-energies in the Brueckner ladder approximation. Several attempts have recently been made to improve the first two approximations ⁶. They are expected to be important e.g. for subthreshold particle production, but it is not known whether they are also relevant for flow observables.

With the above approximations one arrives at a BUU-type transport equation with DB self-energies which should be specified for the general non-equilibrium configuration. Since these cannot be calculated one further introduces a local nuclear matter approximation neglecting spatial variations

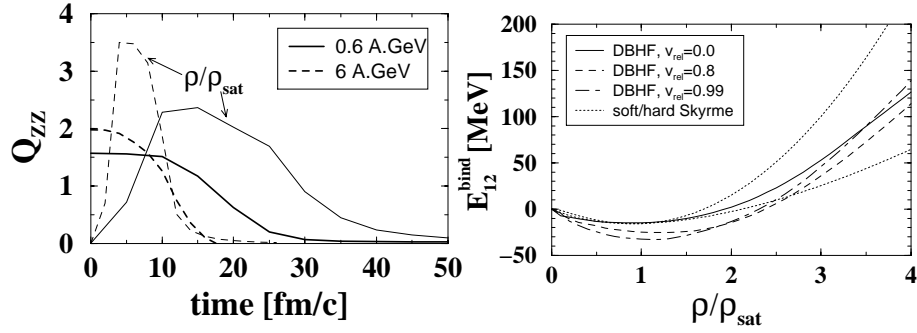


Figure 1. Left: Time evolution of the quadrupole moment of the energy-momentum tensor Q_{zz} (bold lines) and of the central density (in units of ρ_{sat} , dashed lines) at the center in central $Au + Au$ collisions at 0.6 and 6.0 A.GeV. Right: Energy per particle of a two-Fermi-sphere configuration (CNM) (minus the relative kinetic energy) as a function of total density. This “reduced” EOS is shown for DB self energies for several relative velocities and for two (equilibrium) Skyrme parametrizations.

and adopting a model for the local nuclear matter phase space distribution. Two models have mainly been used: One is the Local Density Approximation (LDA) assuming equilibrated nuclear matter at the local total density, usually supplemented with an empirical momentum dependence taken from the real part of the optical model. The other is the Local phase space Configuration Approximation (LCA) which assumes a configuration of two interpenetrating streams of nuclear matter with a given relative velocity (also called colliding nuclear matter, CNM), i.e. a two-Fermi-sphere configuration with Pauli-effects taken into account. In the first case this is directly the nuclear EOS. For the second case the self energies have not been calculated in the DB approach (but for the non-relativistic case ⁷). However, we have developed a self-consistent extrapolation starting from nuclear matter DB calculations as discussed in detail in ref. ⁸. This then gives the model non-equilibrium self energies as a function of the configuration parameters, namely the two Fermi momenta and the relative velocity.

The relevance of such non-equilibrium effects in the dynamical situation of heavy ion collisions is demonstrated in Fig. 1 (left side). Here the quadrupole moment of the energy-momentum tensor $Q_{zz} = \frac{2T^{33} - T^{11} - T^{22}}{T^{33} + T^{11} + T^{22}}$ is shown together with the baryon density as a function of time. It is clearly seen that the local phase space is significantly anisotropic during times which are comparable with the compression phase of the collision independent of beam energy. Hence, non-equilibrium effects are present during a large part of the compression phase.

sion phase where one wants to study the EOS at high densities. Therefore non-equilibrium effects should be considered at least on the level of the effective in-medium interaction, i.e. the mean fields in transport calculations should be taken for non-equilibrium configurations instead for equilibrated nuclear matter. Actual transport calculations have verified that the anisotropic phase space in heavy ion reactions can be parametrized reliably by CNM configurations⁹.

In order to have a qualitative feeling for the non-equilibrium effects we define an effective, “reduced” EOS for the CNM configuration. This is given by subtracting from the binding energy per particle for the CNM configuration with a given relative velocity v_{rel} the irrelevant kinetic energy of relative motion between the two streams. This quantity is shown on the right side of Fig. 1 for symmetric CNM configurations ($k_{F_1} = k_{F_2}$) for relative velocities $v_{rel} = 0.8(0.99)$ corresponding to beam energies of 0.6(6.0) GeV per nucleon as a function of the total density. $v_{rel} = 0$ corresponds to the isotropic case (equilibrated nuclear matter, i.e. one Fermisphere). As a reference we also show the widely used Skyrme parameterisations with a soft/hard EOS with $K = 200/380$ MeV¹.

It is seen that the effective EOS becomes more attractive, i.e. softer, compared to the equilibrium EOS and saturates at a higher total density. This effect is understood qualitatively in the following way: If there was no interaction between the two nuclear matter streams then the reduced energy per particle of the two-Fermi-sphere configuration were just the energy per particle of one Fermi sphere at half the total density. The minimum of the reduced EOS would then be shifted to twice ρ_{sat} . The interaction energy per particle can be estimated as twice the real part of the optical potential at the relative momentum which can be estimated as twice the Fermi momentum at the corresponding density. For saturation density this is attractive but decreases with increasing momentum. Thus we expect a minimum of the reduced EOS between one and twice ρ_{sat} and below the saturation energy of equilibrium nuclear matter. This is what is seen qualitatively in Fig. 1.

The question of interest is now whether these non-equilibrium effects are observable in terms of collective flow. We have therefore performed transport calculations using CNM self energies derived from DB mean fields calculated by the Tübingen-group (DBF)¹⁰ in the two approximations: the LDA which refers to equilibrated nuclear matter (DBF/LDA), and the local configuration approximation (LCA) where the non-equilibrium DBF mean fields are used in the transport calculations (DBF/LCA).

We discuss azimuthal distributions as function of transverse momentum, rapidity, centrality and beam energy. Such distributions have been

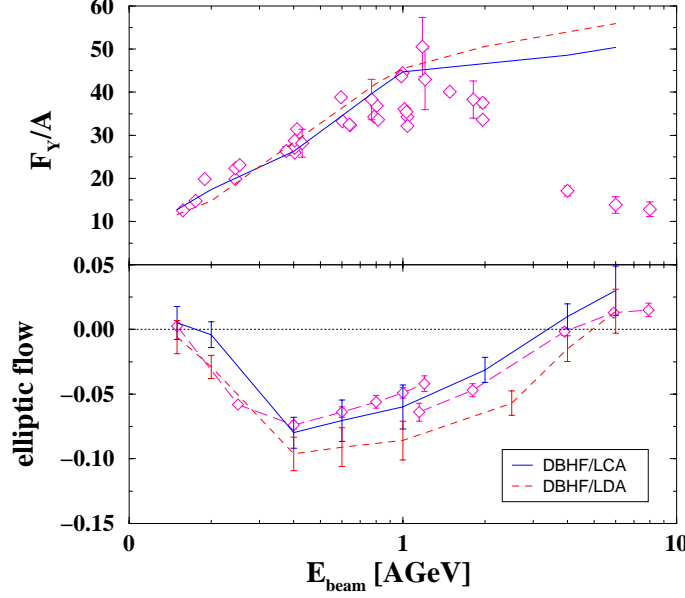


Figure 2. Flow per particle F_y/A and elliptic flow v_2 at mid-rapidity ($|\Delta Y^{(0)}| < 0.1$) in semi-central $Au + Au$ reactions from SIS to AGS energies. The transport calculations ¹¹ with microscopic DB mean fields in the LCA (solid) and LDA (dashed) approaches are compared to the data (symbols) ^{2,4}.

parametrized in terms of a Fourier series $dN/d\phi = a_0(1 + 2a_1\cos(\phi) + 2a_2\cos(2\phi))$ with ϕ the azimuthal angle with respect to the reaction plane. The coefficients a_1 and a_2 describe the in-plane and out-of-plane components of collective flow, respectively, and they depend on transverse momentum $P_t^{(0)} = (P/A)/(P_{cm}^{proj}/A_{proj})$, rapidity $Y^{(0)} = Y_{cm}/Y_{cm}^{proj}$, centrality and energy. The in-plane or directed flow v_1 can also be characterized by the

quantity $F_y = \left. \frac{d\langle k_x(Y^{(0)}) \rangle / A}{dY^{(0)}} \right|_{Y^{(0)}=0}$ as the slope of the mean in-plane trans-

verse momentum at mid rapidity. v_2 is often referred to as the elliptic flow. Its sign describes the transition from out-of-plane ($v_2 < 0$) to in-plane flow ($v_2 > 0$) ⁴. The squeeze-out ratio R_N is connected to the elliptic flow through $R_N = (1 - 2v_2)/(1 + 2v_2)$. Fig. 2 shows the observables F_y/A and v_2 for $Au + Au$ collisions from SIS to AGS energies. Rather complete data are available from the EOS and FOPI collaborations, which are, however, not completely consistent for v_2 . It is seen that the calculations ¹¹ reproduce the flow F_y only for SIS energies, while at AGS energies the calculations overesti-

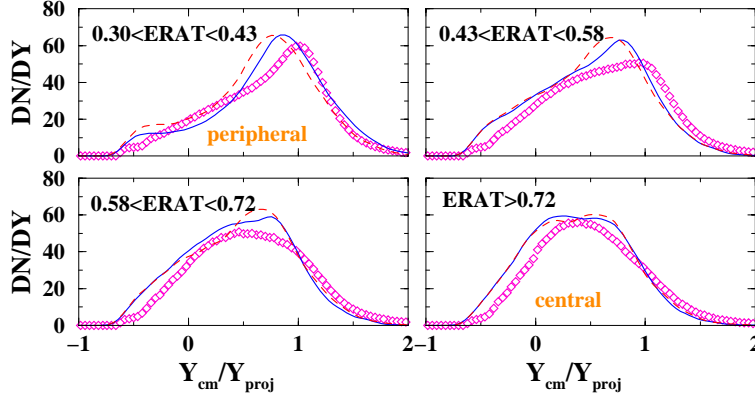


Figure 3. Rapidity distributions for $Au + Au$ collisions at 400 AMeV selected according to bins of ERAT which is a measure of centrality (increasing with increasing centrality). Calculations are shown for the LCA (solid) and LDA (dashed) approximations for the self energies and compared with the data (symbols) ¹³.

mate the data. This is to be expected because the DB self energies do not reproduce the empirical saturation of the real part of the optical potential at energies above about 1 GeV, but rather lead to linearly increasing repulsion. This situation also appears in non-relativistic descriptions using momentum dependent Skyrme-parametrizations (see ¹²). The difference between the LDA and LCA descriptions is not significant because the DBF self energies are not strongly momentum dependent at saturation density.

The description of the elliptic flow is generally much better because as a ratio it is not so strongly dependent on the absolute transverse flow. We see that both models generally reproduce the qualitative trend of the data, i.e. the transition from small v_2 at 0.2 A.GeV to a preferential out-of plane flow ($v_2 < 0$) which is maximal at 0.4 A.GeV with the subsequent transition back to in-plane flow around 4 A.GeV. There is now, however, a pronounced difference between the two approximations. The calculation without non-equilibrium effects (LDA) shows a larger squeeze-out effect and also has a larger transition energy from a out-of-plane to an in-plane emission of participant matter. This effect is reduced for the LCA calculation which can be understood by the effective softening of the equation of state during the initial non-equilibrium phase of the collision as discussed in connection with Fig. 1. We also mention that the observed difference is of the same magnitude as the difference between a soft and hard EOS (see Fig. 2 of Ref. ¹²). Furthermore the inclusion of the non-equilibrium effects considerably improves the agreement with the data

which means that the underlying effective interaction (DB) is able to describe the dynamics of heavy ion collisions.

In Fig. 3 we show rapidity distributions for $Au + Au$ collisions at 400 AMeV selected according to a quantity ERAT, the ratio between transverse and longitudinal total momentum, which has been shown to be a measure of centrality¹³. The rapidity distributions are reasonably well reproduced, in particular for central collisions (large ERAT). A general tendency is that the spectator particles observed at $Y^{(0)} \simeq 1$ in the experiment are more stopped in the calculations. This effect is larger in LDA than LCA, because the field is more repulsive in the former case (see Fig. 1).

To summarise the effective equation-of-state probed by the compression phase in energetic heavy ion reactions is in a significant way governed by local non-equilibrium. The anisotropy of the phase space lowers the binding energy per particle and makes the effective EOS seen in heavy ion reactions softer. This fact is reflected in the behaviour of collective flow phenomena. We conclude that “geometric” phase space effects should be taken into account on the level of the effective interaction when conclusions on the equilibrium EOS are drawn from heavy ion collisions. The comparison with experiments shows that microscopic many-body methods that are successful for nuclear structure also explain heavy ion reactions quantitatively at SIS energies¹⁴.

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